BAYESIAN ESTIMATION OF RELIABILITY

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BAYESIAN ESTIMATION OF RELIABILITY

DEPRIWANA RAHMI

A dissertation submitted in fulfilment
Of the requirements for the award of the Degree of
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November 2005

"I declare that this dissertation entitled '*Bayesian Estimation of Reliability*' is the result of my own research except as cited in the references. The dissertation has not been accepted for any degree and is not concurrently submitted in candidature of any other degree"

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Alhamdullillah..

This dissertation is dedicated to my beloved Papa Darsamin and Mama Ernawati,
Thanks for supporting me
Brothers and sisters;
Thanks for encouraging and supporting me and also to someone who
Always by my side, Bang Teddy Purnamirza, thanks for all.

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ABSTRACT

Most of statistical theory thought at the undergraduate level is based on classical probability or frequentist probability. Also, the estimation techniques widely used are maximum likelihood estimation (MLE) and methods of moment. This dissertation discusses the subjective probability as an alternative theory, where Bayesian estimation is used. Based on a random sample of 20 observations that was generated from Weibull distribution, we compare the reliability estimation using MLE and Bayesian approach. We found out that the mean percentage error for Bayesian is 7.9678, which is lower than MLE mean percentage error of 10.8434. Thus, bayesian reliability estimation is better.

ABSTRAK

Kebanyakan teori statistik yang diajar pada peringkat sarjana muda adalah berdasarkan kebarangkalian klasik atau kebarangkalian frekuensi. Teknik anggaran yang biasa digunakan dengan meluas ialah kebolehjadian maksimum dan kaedah momen. Oleh itu, disertasi ini akan membincangkan tentang kebarangkalian subjektif sebagai teori alternatifnya, yang menggunakan anggaran Bayesian. Berdasarkan sampel rawak, 20 cerapan yang diambil daripada taburan Weibull, anggaran kebolehpercayaan menggunakan anggaran kebolehjadian maksimum dengan pendekatan Bayes dibandingkan. Didapati min peratusan ralat bagi Bayes adalah 7.9678. Nilai ini lebih rendah berbanding dengan anggaran kebolehjadian maksimum yang memberikan nilai 10.8434. Dengan itu anggaran kebolehpercayaan Bayes adalah lebih baik.

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LIST OF SYMBOLS

K	:	Constant
N	:	Sample size
x	:	Random Variables
β	:	Shape Parameter
α	:	Scale Parameter
β^*	:	Shape Parameter of Bayesian Estimation
α^*	:	Scale Parameter of Bayesian Estimation
$\hat{\beta}$	:	Shape Parameter of Maximum Likelihood Estimation
$\hat{\alpha}$	:	Scale Parameter of Maximum Likelihood Estimation
$F(t)$	:	Cumulative Distribution
$S(t)$	:	Survival Function
$R(t)$	:	Reliability Function
$R^*(t)$	:	Reliability Estimation Of Bayesian
$\hat{R}(t)$	:	Reliability Estimation of MLE

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LIST OF ABBREVIATIONS

PDF	-	Probability Density Function
CF	-	Cummulative Function
MLE	-	Maximum Likelihood Estimation

CHAPTER 1

RESEARCH FRAMEWORK

1.1 Background and Motivation

In real world problem, we often face with the task of making inferences or decisions under uncertainty. The uncertainty of any event may be represented quantitatively in the form of probability. There are four types of probability theory:

1. *Classical probability*

Classical logic is the most important formalism to philosophers of science, but is limited because propositions can only be true or false, and nothing between. The classical interpretation of probability is based on the concept of *equally likely outcomes* that is has the same probability (DeGroot H, 1986).

Two basic difficulties arise when an attempt is made to develop a formal definition of probability from the classical interpretation (DeGroot H, 1986)**.**

1. The concept of equally likely outcomes is essentially based on the concept of probability that we are trying to define. The statement that two possible outcomes are equally likely to occur is the same as the statement that two outcomes have the same probability.
2. No systematic method is given for assigning probabilities to outcomes that are not assumed to be equally likely.

2. Logical probability

The probability of an event is the rational degree of belief in the event relative to some given evidence. But this interpretation of probability has been generally abandoned because prescription was never given for how logical probabilities were to be assigned and compared (Popper, 1959).

3. Frequency probability

The frequency or frequentist probability of an event is a factual statement about the world. If an event can occur in just a finite number of ways, the frequency probability of the event is defined as the ratio of the favorable number of ways the particular event can occur compared with the total number of possible event (James, 2003).

4. *Subjective probability*

The subjective probability is based on the natural concept of probability, as degree of belief, related to a status of uncertainty, whilst the Bayes' theorem is the logical tool to update the probability in the light of new pieces of information (D'Agostini, 2003) These theories are different from logical theories in that, different individuals are expected to have different probabilities for an event, even when their knowledge is the same.

Different events may have different degree of probability, depending whether we think that they are more likely to be true or false. Different expressions could be used to mean exactly the same concept. For example, given two events E_1 and E_2, it could be said: E_2 is considerable more likely than E_1; E_2 is more confident; having to choose between E_1 and E_2 to win a price, E_2 would be promptly chosen (D'Agostini, 2003).

It is reported that basic difference between Bayesian and frequentist is these Bayesians condition on the data actually observed, and consider the probability distribution on the hypothesis; Bayesians believe it is reasonable to put probability distribution on hypothesis and they behave accordingly (Jeffrey, 1961).

While the frequentist condition on a hypothesis of choice and consider the probability distribution on the data, whether observed or not, frequentists do not think it is reasonable to put probability distributions on hypothesis (in their opinion, one hypothesis is true, the rest are false, even we do not know which is the case). Maximum Likelihood Estimation (MLE) is the estimation technique that is used in frequentist probability.

Bayesians are interested in the way this function varies with parameters. While the frequentists are interested in its variation on the data. The frequentist is that it is never talked about the probability distribution of the true value of a parameter. The Bayesian posterior uses the marginal distribution at each point, integrate out the nuisance parameter, the frequentist uses the conditional probability.

Maximum Likelihood Estimation (MLE) is generally used in many applications as an estimation technique for a particular data as well as the others technique such as Method of Moment, Least Squared, etc. Meanwhile, the Bayesian technique is rarely used as an estimation technique for a particular data. Therefore, it is interesting and challenging to implement the Bayesian technique as an estimation technique for a particular data as well as analyzing the result between Bayesian estimation and Maximum Likelihood Estimation.

1.2 Research Objectives and Scopes

The main objective of this dissertation is to conduct the estimation of weibull scale and shape parameters using Bayesian method and non-Bayesian method. A reliability estimation using these techniques are compared as an application these techniques.

In this dissertation, the estimation result between Bayesian technique and maximum likelihood estimation will be analyzed. The reliability of Bayesian will also be analyzed.

The maximum likelihood estimation (MLE) will be used as estimation technique for non-Bayesian Method. The reason why MLE is used is because MLE is the estimation technique of frequentist probability.

Since it is difficult to calculate all the formulas in chapter 3 manually, the Matlab programming is written and run to calculate all the formulas and obtain the graph.

1.3 Organization of Dissertation

This dissertation is organized into four chapters.

Chapter 1 explains the research framework. It begins with the explanation of the background of the dissertation, and the explanation of the objectives and scope of this study.

Chapter 2 concludes the relevant literature on Bayesian estimation, non-Bayesian estimation, reliability analysis and their applications.

Chapter 3 calculates and analyzes the result of Bayesian estimation and maximum likelihood estimation (MLE). The analysis of the reliability of the Bayesian estimation is also presented.

Chapter 4 summarizes the whole study. This chapter includes the conclusions and the suggestions for the future research.

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